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Determination of the Secular Perturbations of Minor Planet Ceres, arising from the actions of the eight Major Planets of the Solar System. By C. J. Merfield.

Introduction.—So far as the writer knows, there has been no complete determination of the secular perturbations of the minor planet Ceres.

Dr. Hill has published * a method of computing absolute perturbations, and as an example of the method he calculated † the secular perturbations of this asteroid in the cases of three of the major planets, Mars, Jupiter, and Saturn, the osculating elements by Schubert being adopted as a foundation of his work.

A system of Mean Elements of Ceres being now available, the writer was induced to undertake the complete determination of the secular perturbations of this body, as a contribution to the science of astronomy that seemed worthy of undertaking.

Elements.—The elements of the major planets, adopted in this work, are those given by Dr. Hill in his investigation ‡ of “A New Theory of Jupiter and Saturn.” The Mean Elements of Ceres are by the same investigator, and are probably not very far from the true ones; no doubt, upon the whole, they will require very slight amendment, and the alterations necessary to bring about a better representation of the normal places will not influence the results to an extent requiring consideration.

* *Astronomische Nachrichten*, 1982.

† *Astronomical Journal*, 368.

‡ *Astronomical Papers, prepared for the use of the American Ephemeris and Nautical Almanac*, vol. iv. The masses of Venus, Earth, and Uranus adopted by Hill are 1/408134, 1/327000, 1/22640.

The values of the secular perturbations of Ceres, deduced from this inquiry, may be accepted as definitive if the preceding premise be found correct.

Methods of Calculation.—The method* of Gauss has been adopted, as explained by Dr. Hill. The values of the integrals, involved in the formulæ, have been interpolated from the table appended to his memoir. As the moduli in all cases came within the limits of the arguments given, this table was of much value, and considerably reduced the numerical work.

The values of the functions were deduced by the aid of seven figure logarithms; and, where it was advantageous, addition and subtraction logarithms, to a similar degree of accuracy, were used.

As the work proceeded, certain well-known checks were applied, and in all instances they were completely satisfied. The accuracy of the perturbations in the plane of the orbit has been verified by the usual formula; the residual is given in each case as an indication of the exactness of the calculation.

The equality of the summations of the functions for the odd and even divisions forms a useful control against large numerical errors, but it is not infallible, owing to certain circumstances arising in each special case.

Consulting the volumes, *locis citatis*, the significance of the notation will be found. The results of the determination are therefore given without further comment. Just so much of the numerical details are appended as will form a guide to a future investigation, and the detection of any errors that may have been made.

ELEMENTS OF CERES.

The following are the Mean Elements determined by Dr. Hill, and used as the basis of the investigation here presented.

Elements.

$$n = 281504''850309$$

$$\text{Log } e = 8.89458493$$

$$\left. \begin{array}{rcl} \pi & = & 148^\circ 28' 32''.5 \\ \Omega & = & 80^\circ 48' 5''.6 \\ i & = & 10^\circ 37' 6''.2 \end{array} \right\} 1850$$

$$\text{Log } a = 0.4420738$$

Dividing the orbit into sixteen equal parts, with respect to the eccentric anomaly, the following values of the true anomalies and logarithms of the radii vectores are deduced.

* *Gauss's Method of Computing Secular Perturbations: Astronomical Papers, prepared for the use of the American Ephemeris*, vol. i. part v.

<i>n</i>	E	<i>v</i>			Log <i>r</i>	<i>n</i>	E	<i>v</i>			Log <i>r</i>
1	0°0	0	0	0°00	0·4065934	9	180°0	180	0	0°00	0·4748732
2	22°5	24	17	14·89	0·4093985	10	202°5	200	50	15·83	0·4724618
3	45°0	48	16	23·69	0·4172889	11	225°0	221	54	12·95	0·4655203
4	67°5	71	43	14·16	0·4288362	12	247°5	243	24	15·50	0·4549198
5	90°0	94	29	57·82	0·4420738	13	270°0	265	30	2·18	0·4420738
6	112°5	116	35	44·50	0·4549198	14	292°5	288	16	45·84	0·4288362
7	135°0	138	5	47·05	0·4655203	15	315°0	311	43	36·31	0·4172889
8	157°5	159	9	44·17	0·4724618	16	337°5	335	42	45·11	0·4093985

We also have the constant quantities used throughout the calculation—

$$\omega = 67^{\circ} 40' 26''.90$$

$$\phi = 4^{\circ} 29' 57''.82$$

$$\text{Log sin } \phi = 8.8945849$$

$$\text{Log sin } i = 9.2654471$$

$$,, \text{ cos } \phi = 9.9986595$$

$$,, \text{ cos } i = 9.9924988$$

$$\text{Log sin}^2 \frac{\phi}{2} = 7.1877800$$

$$\text{Log sin}^2 \frac{i}{2} = 7.9325686$$

I. MERCURY.

Elements.

$$n' = 5381016''.260$$

$$\text{Log } e' = 9.31303314$$

$$\left. \begin{array}{l} \pi' = 75^{\circ} 7' 13''.62 \\ \Omega' = 46 33 8.63 \\ i' = 7 0 7.71 \end{array} \right\} 1850$$

$$\text{Log } a' = 9.5878217$$

$$m' = 1/7500000$$

$$I = 6^{\circ} 13' 11''.06$$

$$K = 72^{\circ} 49' 16''.31$$

$$II = 208 23 0.32$$

$$K' = 73 9 33.89$$

$$II' = 135 23 35.24$$

$$\text{Log } k = 9.9987395$$

$$\text{Log } C = 7.8017097$$

$$,, \text{ } k' = 9.9987043$$

Secular Perturbations.

$$\left[\frac{de}{dt} \right]_{00} = -0.000020 \quad \left[\frac{d\Omega}{dt} \right]_{00} = -0.000257$$

$$\left[\frac{d\chi}{dt} \right]_{00} = +0.000521 \quad \left[\frac{d\pi}{dt} \right]_{00} = +0.000517$$

$$\left[\frac{di}{dt} \right]_{00} = +0.000047 \quad \left[\frac{dL}{dt} \right]_{00} = +0.076247$$

$$A_1^{(s)} \sin \phi + B_0^{(c)} \cos \phi = -0.0000000013$$

II. VENUS.

Elements.

$$n' = 2106641.357$$

$$\text{Log } e' = 7.83525353$$

$$\left. \begin{array}{l} \pi' = 129^\circ 27' 42''.83 \\ \Omega' = 75^\circ 19' 53''.08 \\ i' = 3^\circ 23' 35''.01 \end{array} \right\} 1850$$

$$\text{Log } a' = 9.8593378$$

$$m' = 1/425000$$

$$I = 7^\circ 14' 52''.52$$

$$K = 19^\circ 12' 48''.63$$

$$II = 245^\circ 6' 39''.72$$

$$K' = 18^\circ 45' 15''.08$$

$$II' = 226^\circ 7' 37''.99$$

$$\text{Log } k = 9.9981963$$

$$\text{Log } C = 5.3891827$$

$$,, \quad k' = 9.9983334$$

Secular Perturbations.

$$\left[\frac{de}{dt} \right]_{00} = -0.000025 \quad \left[\frac{d\Omega}{dt} \right]_{00} = -0.026580$$

$$\left[\frac{d\chi}{dt} \right]_{00} = +0.037022 \quad \left[\frac{d\pi}{dt} \right]_{00} = +0.036567$$

$$\left[\frac{di}{dt} \right]_{00} = +0.000219 \quad \left[\frac{dL}{dt} \right]_{00} = +1.395618$$

$$A_1^{(s)} \sin \phi + B_0^{(c)} \cos \phi = +0.0000000004$$

Owing to certain circumstances, a verification of certain functions was undertaken. The orbit of Ceres was again divided into another sixteen parts, and the result of the investigation fully established the correctness of the previous calculation. The summations of the various functions, denoted by Σ_3 , are given below the tabular statement, for comparison with $\Sigma_1 + \Sigma_2$. The check on the perturbations in the plane of the orbit in this instance gives a residual—

$$-0.000000004$$

The above results are deduced from the several functions of the first investigation.

III. EARTH.

Elements.

$$n' = 1295977''.416$$

$$\text{Log } e' = 8.22456263$$

$$\left. \begin{array}{l} \pi' = 100^\circ 21' 39''.73 \\ \Omega' = \text{---} \text{---} \text{---} \\ i' = 0 \quad 0 \quad 0.00 \end{array} \right\} 1850$$

$$\text{Log } a' = 0.00000000$$

$$m' = 1/322800$$

$$I = 10^\circ 37' 6''.20$$

$$K = 48^\circ 25' 29''.13$$

$$II = 247^\circ 40' 26''.90$$

$$K' = 47^\circ 48' 1''.33$$

$$II' = 199^\circ 33' 34''.13$$

$$\text{Log } k = 9.9991720$$

$$\text{Log } C = 6.4491253$$

$$,, \quad k' = 9.9933526$$

Secular Perturbations.

$$\left[\frac{de}{dt} \right]_{00} = -0.000547$$

$$\left[\frac{d\Omega}{dt} \right]_{00} = -0.108987$$

$$\left[\frac{d\chi}{dt} \right]_{00} = +0.096102$$

$$\left[\frac{d\pi}{dt} \right]_{00} = +0.094236$$

$$\left[\frac{di}{dt} \right]_{00} = +0.000011$$

$$\left[\frac{dL}{dt} \right]_{00} = +1.926030$$

$$A_1^{(s)} \sin \phi + B_0^{(c)} \cos \phi = -0.000000009$$

IV. MARS.

Elements.

$$n' = 68905''\cdot784$$

$$\text{Log } e' = 8\cdot96973284$$

$$\left. \begin{array}{l} \pi' = 333^{\circ} 17' 51''\cdot74 \\ \Omega' = 48^{\circ} 23' 54''\cdot59 \\ i' = 1^{\circ} 51' 2''\cdot24 \end{array} \right\} 1850$$

$$\text{Log } a' = 0\cdot1828971$$

$$m' = 1/3093500$$

$$I = 9^{\circ} 6' 33''\cdot95$$

$$K = 175^{\circ} 21' 16''\cdot67$$

$$II = 241^{\circ} 23' 55''\cdot82$$

$$K' = 174^{\circ} 49' 10''\cdot35$$

$$II' = 66^{\circ} 18' 46''\cdot43$$

$$\text{Log } k = 9\cdot9953869$$

$$\text{Log } C = 8\cdot3052599$$

$$,, k' = 9\cdot9991198$$

Secular Perturbations.

$$\left[\frac{de}{dt} \right]_{00} = +''\cdot0000068$$

$$\left[\frac{d\Omega}{dt} \right]_{00} = -''\cdot039884$$

$$\left[\frac{d\chi}{dt} \right]_{00} = +''\cdot064691$$

$$\left[\frac{d\pi}{dt} \right]_{00} = +''\cdot064008$$

$$\left[\frac{di}{dt} \right]_{00} = +''\cdot000357$$

$$\left[\frac{dL}{dt} \right]_{00} = +''\cdot238781$$

$$A_1^{(s)} \sin \phi + B_0^{(c)} \cos \phi = +0\cdot0000000007$$

Owing to the inequality of the quantities Σ_1 and Σ_2 , for several of the functions, it was decided to divide the orbit of Ceres into another sixteen equal parts and make a separate calculation.

The results of the summations of the several functions in each case are given below the tabular statement; they are denoted by Σ_g . A comparison of these, with the quantity $\Sigma_1 + \Sigma_2$, completely verifies the accuracy of the first investigation.

The secular perturbations given above are determined from the first computed coefficients, as no alteration would take place by combining the results for the thirty-two divisions. The residual of the check equation for testing the accuracy of the perturbations in the plane of the orbit for the second division is $+0\cdot000000002$.

V. JUPITER.

Elements.

$$n' = 109256''.62552$$

$$\text{Log } e' = 8.68354330$$

$$\left. \begin{array}{l} \pi' = 11^\circ 54' 31''.67 \\ \Omega' = 98^\circ 56' 19''.79 \\ i' = 1^\circ 18' 42''.10 \end{array} \right\} 1850$$

$$\text{Log } a' = 0.7162374$$

$$m' = 1/1047.879$$

$$I = 9^\circ 22' 50''.37$$

$$K = 136^\circ 19' 14''.71$$

$$II = 250^\circ 10' 47''.57$$

$$K' = 136^\circ 53' 10''.58$$

$$II' = 113^\circ 34' 30''.27$$

$$\text{Log } k = 9.9950989$$

$$\text{Log } C = 8.7995614$$

$$,, \quad k' = 9.9990752$$

Secular Perturbations.

$$\left[\frac{de}{dt} \right]_{00} = - 0''.675238$$

$$\left[\frac{d\Omega}{dt} \right]_{00} = - 52''.157759$$

$$\left[\frac{d\chi}{dt} \right]_{00} = + 56''.772540$$

$$\left[\frac{d\pi}{dt} \right]_{00} = + 55''.879400$$

$$\left[\frac{di}{dt} \right]_{00} = - 0''.577398$$

$$\left[\frac{dL}{dt} \right]_{00} = - 56''.026115$$

$$A_1^{(s)} \sin \phi + B_0^{(c)} \cos \phi = - 0.000000001$$

VI. SATURN.

Elements.

$$n' = 43996''.21506$$

$$\text{Log } e' = 8.74865503$$

$$\left. \begin{array}{l} \pi' = 90^\circ 6' 41''.37 \\ \Omega' = 112^\circ 20' 40''.05 \\ i' = 2^\circ 29' 40''.19 \end{array} \right\} 1850$$

$$\text{Log } a' = 0.97949564$$

$$m' = 1/3501.6$$

$$I = 8^{\circ} 35' 27''.85$$

$$K = 58^{\circ} 40' 26''.61$$

$$II = 256 26 31.11$$

$$K' = 58 17 41.41$$

$$II' = 197 57 23.97$$

$$\text{Log } k = 9.9995389$$

$$\text{Log } C = 9.4563013$$

$$,, \quad k' = 9.9955700$$

Secular Perturbations.

$$\left[\frac{de}{dt} \right]_{00} = -0''.030004$$

$$\left[\frac{d\Omega}{dt} \right]_{00} = -1''.410009$$

$$\left[\frac{d\chi}{dt} \right]_{00} = +1''.339545$$

$$\left[\frac{d\pi}{dt} \right]_{00} = +1''.315400$$

$$\left[\frac{di}{dt} \right]_{00} = -0''.040972$$

$$\left[\frac{dL}{dt} \right]_{00} = -2''.122041$$

$$A_1^{(s)} \sin \phi + B_0^{(c)} \cos \phi = -0.000000007$$

VII. URANUS.

Elements.

$$n' = 15425''.752$$

$$\text{Log } e' = 8.67139132$$

$$\left. \begin{array}{l} \pi' = 168 15 6.70 \\ \Omega' = 73 14 8.00 \\ i' = 0 46 20.54 \end{array} \right\} 1850$$

$$\text{Log } a' = 1.2831044$$

$$m' = 1/22800$$

$$I = 9^{\circ} 51' 11''.74$$

$$K = 340^{\circ} 15' 40''.28$$

$$II = 247 4 47.40$$

$$K' = 340 10 5.55$$

$$II' = 266 51 55.71$$

$$\text{Log } k = 9.9935658$$

$$\text{Log } C = 9.9089914$$

$$,, \quad k' = 9.9999810$$

Secular Perturbations.

$$\left[\frac{de}{dt} \right]_{00} = +0''.000195$$

$$\left[\frac{d\Omega}{dt} \right]_{00} = -0''.026973$$

$$\left[\frac{d\chi}{dt} \right]_{00} = + 0''.023392$$

$$\left[\frac{d\pi}{dt} \right]_{00} = + 0''.022930$$

$$\left[\frac{di}{dt} \right]_{00} = + 0''.000000,2$$

$$\left[\frac{dL}{dt} \right]_{00} = - 0''.036902$$

$$A_1^{(s)} \sin \phi + B_0^{(c)} \cos \phi = + 0.000000000002$$

VIII. NEPTUNE.

Elements.

$$n' = 7864''.935$$

$$\text{Log } e' = 7.9292247$$

$$\left. \begin{array}{l} \pi' = 43^\circ 17' 30''.30 \\ \Omega' = 130^\circ 7' 31''.83 \\ i' = 1^\circ 47' 1''.68 \end{array} \right\} 1850$$

$$\text{Log } a' = 1.4781414$$

$$m' = 1/19700$$

$$I = 9^\circ 33' 3''.35$$

$$K = 104^\circ 38' 4''.92$$

$$II = 255^\circ 51' 14''.32$$

$$K' = 105^\circ 18' 52''.87$$

$$II' = 150^\circ 52' 49''.92$$

$$\text{Log } k = 9.9985797$$

$$\text{Log } C = 8.8147322$$

$$,, \quad k' = 9.9953889$$

Secular Perturbations.

$$\left[\frac{de}{dt} \right]_{00} = + 0''.000014$$

$$\left[\frac{d\Omega}{dt} \right]_{00} = - 0''.007647$$

$$\left[\frac{d\chi}{dt} \right]_{00} = + 0''.007602$$

$$\left[\frac{d\pi}{dt} \right]_{00} = + 0''.007471$$

$$\left[\frac{di}{dt} \right]_{00} = - 0''.000213$$

$$\left[\frac{dL}{dt} \right]_{00} = - 0''.010975$$

$$A_1^{(s)} \sin \phi + B_0^{(c)} \cos \phi = - 0.000000000001$$

Secular Perturbations of Minor Planet Ceres, arising from the actions of the Eight Major Planets of the Solar System.

Planet	$\left[\frac{de}{dt}\right]_{00}$	$\left[\frac{d\chi}{dt}\right]_{00}$	$\left[\frac{di}{dt}\right]_{00}$	$\left[\frac{d\Omega}{dt}\right]_{00}$	$\left[\frac{d\pi}{dt}\right]_{00}$	$\left[\frac{dL}{dt}\right]_{00}$
Mercury	-0.000020	+0.000521	+0.000047	-0.000257	+0.000517	+0.076247
Venus	-0.000025	+0.037022	+0.000219	-0.026580	+0.036567	+1.395618
Earth	-0.000547	+0.096102	+0.000011	-0.108987	+0.094236	+1.926030
Mars	+0.000068	+0.064691	+0.000357	-0.039884	+0.064008	+0.238781
Jupiter	-0.675238	+56.772540	-0.577398	-52.157759	+55.879400	-56.026115
Saturn	-0.030004	+1.339545	-0.040972	-1.410009	+1.315400	-2.122041
Uranus	+0.000195	+0.023392	+0.000000	-0.026973	+0.022930	-0.036902
Neptune	+0.000014	+0.007602	-0.000213	-0.007647	+0.007471	-0.010975
Summation	-0.705557	+58.341415	-0.617949	-53.778096	+57.420529	-54.559357

Total Secular Perturbations of Ceres.

$$\begin{aligned} \left[\frac{de}{dt}\right]_{00} &= -0.70556 & \left[\frac{d\Omega}{dt}\right]_{00} &= -53.77810 \\ \left[\frac{d\chi}{dt}\right]_{00} &= +58.34142 & \left[\frac{d\pi}{dt}\right]_{00} &= +57.42053 \\ \left[\frac{di}{dt}\right]_{00} &= -0.61795 & \left[\frac{dL}{dt}\right]_{00} &= -54.55936 \end{aligned}$$

Sydney: 1907 March 1.

[The Tables to accompany Mr. Merfield's paper are bound and placed in the Library.]

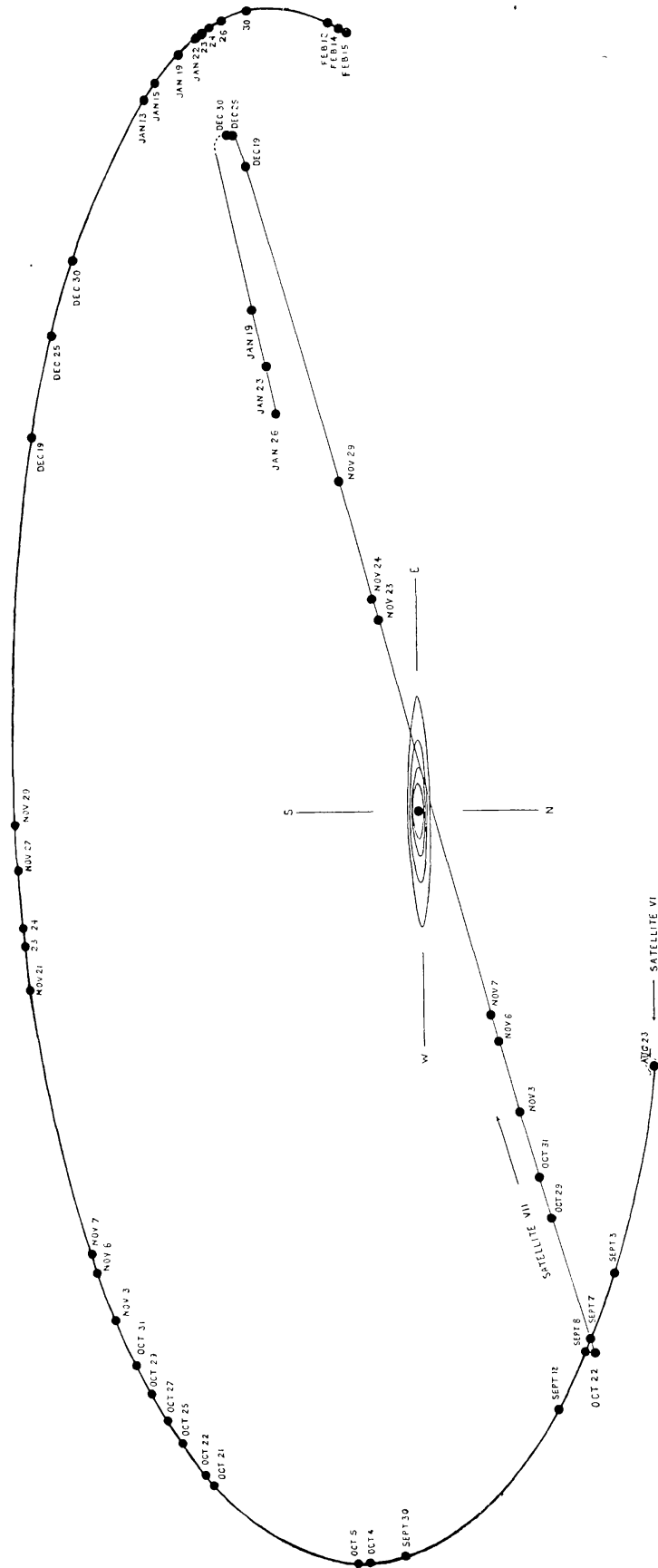


Diagram showing the positions of Jupiter's Satellites VI. and VII., from photographs taken at the Royal Observatory, Greenwich, during the Opposition 1905-6.

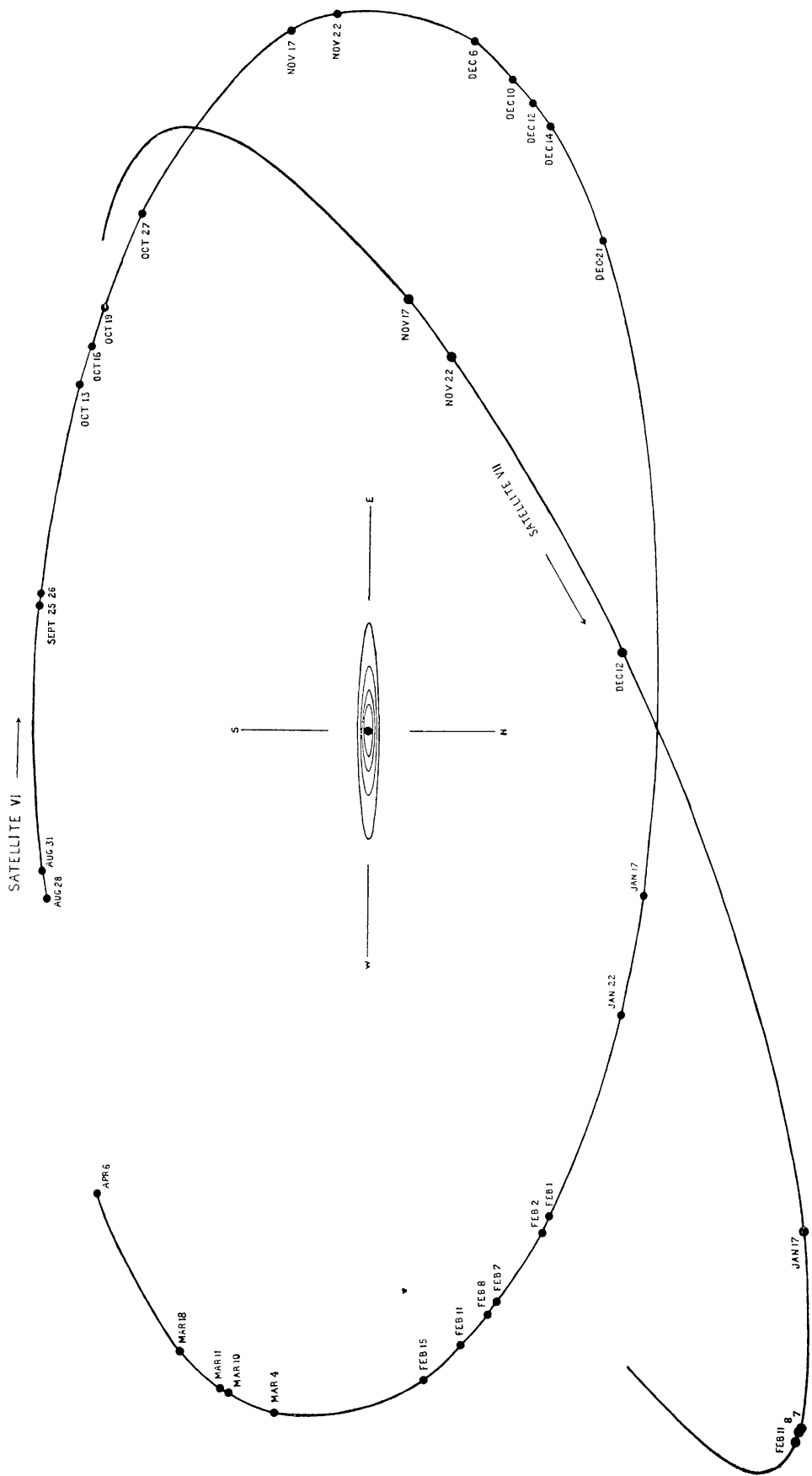


Diagram showing the positions of Jupiter's Satellites VI. and VII., from photographs taken at the Royal Observatory, Greenwich, during the Opposition 1906-7.

Nov. 1907. *Positions of Jupiter's Satellites VI and VII.* 561

Diagrams showing the positions of Jupiter's Satellites VI and VII from Photographs taken at the Royal Observatory, Greenwich, during the Oppositions of 1905-6 and 1906-7. (Plates 4, 5.)

(Communicated by the Astronomer Royal.)

These diagrams are a graphical representation of the observations of Jupiter's Satellites VI and VII from photographs taken at the Royal Observatory with the 30-inch Reflector during the Oppositions of 1905-6 and 1906-7, printed in the *Monthly Notices*, vol. lxvi. p. 438 and vol. lxvii. p. 479.

During the 1905-6 opposition eighty-six photographs of J. VI were secured on thirty-six nights from 1905 August 23 to 1906 February 15—a period of 177 days. Of J. VII nineteen photographs were secured on fifteen nights from 1905 October 22 to 1906 January 26—a period of 97 days.

The opposition 1906-7 was not so favourable as regards weather conditions, but fifty-six photographs of J. VI were obtained on twenty-eight nights over a period of 222 days, and twelve photographs of J. VII on seven nights over a period of 87 days.

The positions plotted in the diagrams are the mean when more than one photograph was taken on any night. The curves were drawn through these points, but in the case of 1906-7 observations of J. VII the curve is continued beyond the observations to indicate the apparent form.

For comparison, the orbits of the four larger satellites are shown, the major axes being plotted to scale, but the minor axes intentionally exaggerated.

Royal Observatory, Greenwich :
1907 September 30.